PROCEEDINGS
OF THE FOURTH INTERNATIONAL COLLOQUIUM ON STRUCTURAL STABILITY MEDITERRANEAN SESSION

SEPTEMBER 16 - 20, 1991
ISTANBUL

EDITOR
GÜLAY AŞKAR

Load Carrying Capacity of Beams Subjected to Local Plate Buckling and Overall Lateral Torsional Buckling

Prof.Dr.-Ing. J. Lindner
Technical University Berlin, Germany

SUMMARY

Structural members loaded by transverse loading and moments acting in the plane of greatest stiffnes my fail before reaching the full plastic moment by lateral torsional buckling. If the slendernesses of the plated elements flange and web are too great an interaction with local plate buckling may occur. This problem was dealt with by testing on welded I-beams at the Technical University of Berlin.

INTRODUCTION

Different failure modes may occur when beams with I-shape in bending reach their load carrying capacity

i) lateral torsional buckling characterized by a failure mode consisting of a combination of lateral bending and twisting,
ii) plate buckling for individual portions of the cross sections, characterized by deformations rectangular to the plate thickness,
iii) a combination of lateral torsional buckling and plate buckling.

The mutual influence of lateral torsional buckling and plate buckling in the elastic range was investigated by several researchers, (Ref.1), (Ref.2), and comprehensive results were obtained. But normally beams are designed such as to avoid slender beams. The elastic range alone therefore will not cover the whole range of application. Fore that reason the realistic load carrying capacity must also account for the elastic-plastic behaviour of the steel and imperfections, such as out-of straightnesses and residual stresses.

Some theoretical investigations are available (Ref.3). But, especially in the case of complicated theoretical problems, a design procedure must also be checked by means of test results. Only a small number of tests were reported on in the literature, (Ref.4 - 6). Therefore 30 tests were carried out at the Technical University of Berlin in order to investigate the interaction between local plate buckling and overall lateral torsional buckling of beams.

1. TEST SPECIMEN, TEST RIG AND TESTING PROCEDURE

The test specimen are welded beams of mild steel St 37, the nominal cross sections are seen in fig. 1. The actual thicknesses were measured but differed only sligthly from the nominal values. The actual yield strength f_y of the flanges varies between 235 N/mm^2 and 304 N/mm^2, that of the webs between 241 and 364 N/mm^2. These values were determined by coupon tests, loaded with a speed of approximately 10 N/mm^{2}min. Mean values for the cross section properties are given in table 1.

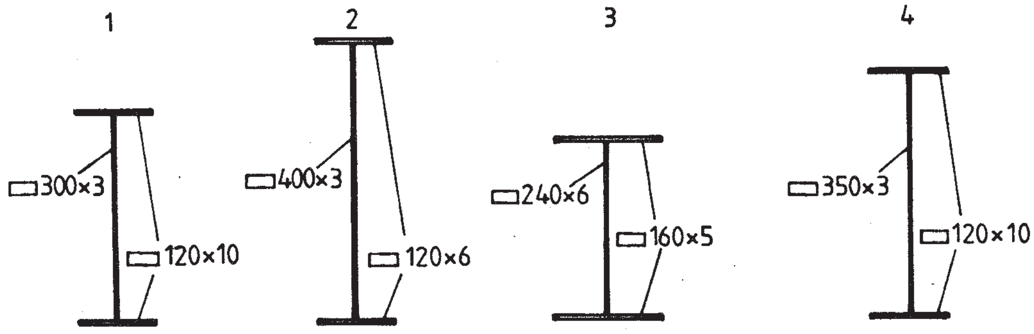

Fig. 1 Cross sections no 1 to 4

Table 1 Mean values for the cross section properties

Cross section	A [cm^2]	I_y [cm^4]	I_z [cm^4]	I_w [cm^6]	I_T [cm^6]	M_{pl} [kNm]
1	26.3	7480	174	71890	2.05	108.8
2	32.8	6300	289	68640	7.96	113.6
3	30.1	3030	336	50330	2.97	74.4
4	34.2	8670	285	91750	8.04	142.4

Cross sections 1 and 4 were loaded by a single load at midspan whereas cross sections 2 and 3 were loaded by uniform bending, each of them with simple support ("fork bearing"). The single load was introduced 10 mm above the top flange with an transverse load eccentricity e. In combination with the measured transverse out of straigthness the total imperfection runs up to of approximately L/1000. The out-of straigthness of the web rectangular to the thickness was relatively great due to welding deformations, the extreme value was h/58, the mean value h/145, with h as the web depth.

The test rig makes sure that the applied load acts always vertical even if the beam deforms laterally and twists. Details were desribed earlier on, (Ref.7).

During the tests deformations were measured. Vertical and horizontal deformations of the top flange and the lower flange were measured at 5 stations along the beams and at 3 stations additionally web deformations were measured.

The load was introduced continuously at a speed of approximately

10 N/mm^{2}min up to 2/3 of the ultimate load and thereafter at semispeed until the maximum load was reached.

The test results are given in table 2. For more detailed information see Ref. 12. The ultimate moments M_u include the self weight of the load introduction construction of the test rig.

Table 2 Test results M_t

test no.	load	cross sec-tion	length L [m]	fail-ure mode	M_t [kNm]	test no.	load	cross sec-tion	length L [m]	fail-ure mode	M_t [kNm]
1A	e	2	2,095	WB	88,4	2A	e	2	2,095	WB	87,6
1B	e	2	2,695	LTB	72,6	2B	e	2	2,695	LTB	71,2
3	e	2	3,695	LTB	62,3	4	e	2	3,695	LTB	56,4
5A	e	2	2,395	LTB	76,8	6A	e	2	2,395	LTB	85,6
5B	e	2	1,690	WB	86,6	6B	e	2	1,690	WB	92,2
7	s	1	2,010	LTB	98,8	8	s	1	2,010	LTB	97,1
9	s	1	2,500	LTB	82,4	11	s	1	2,895	LTB	68,4
13	e	3	3,795	FB	55,3	14	e	3	3,795	FB	57,3
15	e	3	4,893	LTB	50,2	16	e	3	4,893	LTB	46,1
17	e	3	5,180	LTB	46,1	17A	e	3	1,690	FB	75,3
18	e	3	5,180	LTB	48,7	17B	e	3	1,690	FB	76,5
						19	s	4	2,010	LTB	92,3
21	s	4	2,500	LTB	75,3	22	s	4	2,500	LTB	85,8
23	s	4	2,895	LTB	73,3	24	s	4	2,895	LTB	1,9

s single load e endmoment WB web buckling
FB flange buckling LTB lateral torsional buckling

2. GENERAL ASSUMPTIONS FOR TEST EVALUATION

2.1 Proof procedures

The evaluation of the test results is carried out according to the new German stability code DIN 18800, (Ref.9). For thin-walled cross sections the interaction of plate buckling and overall buckling must be taken into account. For this type of cross sections the proof procedures as given in table 3 are provided.

Table 3 Proof procedures

	Procedure	Stress Analysis (actual stress)	Design-Basis (stressability)	max (b/t)-ratios 	
1	elastic-elastic	theory of elasticity	elastic	$0,42 \sqrt{E/f_y}$	$1,28 \sqrt{E/f_y}$
2	elastic-plastic	theory of elasticity	plastic	$0,37 \sqrt{E/f_y}$	$1,25 \sqrt{E/F_y}$

Cross-sections are defined as thinwalled if, for individual portions of the section the max (b/t) ratios are excceded. For constant compression stresses in the individual portion of the cross section these values are given in Fig. 1.

Procedure 1 (elastic-elastic) is the one presently being in common use. Stress-values calculated following the theory of elasticity under γ-factored loads prove that in the highest stressed fibre the limit of yield stress f_y is not being exceeded (γ is the safety coefficient).

For compact sections on the other hand it is allowed to take into account the full plastic capacity of the cross sections. Therefore procedure 2 (elastic-plastic) differs from procedure 1 only in as much, as the plasticicing capacities of the sections are being accounted for. Hereby the sectional resistance is expressed by the full-plastic values N_p, M_p, V_p, but the rotation of the plastic hinge is not allowed for.

In this paper only procedure 2 for thinwalled members is described. Flexural buckling of centrally compressed members or members in compression and bending are not being treated. Here nothing but the solution for the lateral torsional buckling problem of beams in bending is dealt with.

2.2 Effective width of elements

In this model the real width b of a thinwalled element of the cross section is replaced by the effective width b_{ef}.

With regard to the support conditions of the plate elements and on the assumption of elastic behaviour of the cross section (procedure 1 from Fig. 1) the effective width shall be determined from the following formulaes

(a) Stiffened elements

$$b_{ef} / b = 1 \qquad \text{if} \quad \bar{\lambda}_p \leq 0,673 \qquad (1)$$

$$b_{ef} / b = (1-0,22/\bar{\lambda}_p)/\bar{\lambda}_p \qquad \text{if} \quad \bar{\lambda}_p > 0,673 \qquad (2)$$

(b) Unstiffened elements

$$b_{ef} / b = 1 \qquad \text{if} \quad \bar{\lambda}_p \leq 0,7 \qquad (3)$$

$$b_{ef} / b = 0,7/\bar{\lambda}_p \qquad \text{if} \quad \bar{\lambda}_p > 0,7 \qquad (4)$$

Eq.(2) is the well known Winter-formula, (Ref.8). The relative slenderness $\bar{\lambda}$ shall be determined by eq.(5).

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{k \cdot \sigma_e}} = \frac{1,052}{\sqrt{k}} \cdot \frac{b}{t} \sqrt{\frac{f_y}{E}} \qquad (5)$$

where k is the plate buckling coefficient. For plate elements with simple support at both edges or one edge and partially for whole profiles k can be taken from literature.

Using procedure 2 (elastic-plastic) the greatest compressive strain is greater than the strain which leads to first yielding. Thus the effective widths must be reduced additionally, see fig. 2.

where $b^1_{ef} = k_1\ t \sqrt{240/f_y}$, $b^2_{2f} = k_2 \cdot t \sqrt{240/f_y}$

$$b^3_{ef} = b\ \psi_\varepsilon / (\psi_\varepsilon - 1) \qquad (\psi_\varepsilon \le 0) \tag{6}$$

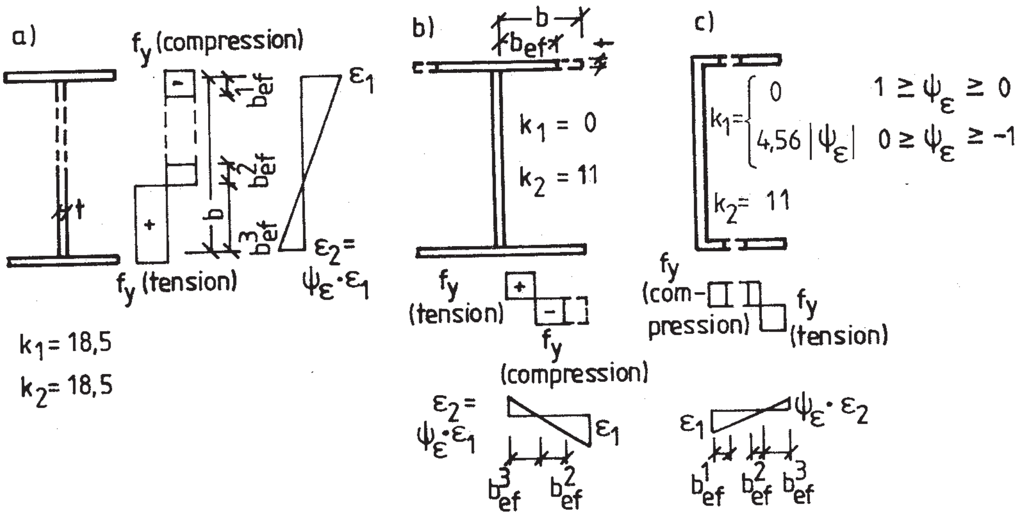

Fig. 2 Definition and distribution of effective width, procedure 2

3. COMPARISON OF TEST RESULTS WITH DESIGN PROCEDURES

3.1 <u>Design procedure A on the basis of a lateral torsional buckling design curve</u>

This design procedure is based on the new German stability code DIN 18800 part 2 (Ref.9), chapter 7, together with DIN 18800 part 3 [15] and the DASt-Richtlinie 016 (Ref.11).

The ultimate moment M_u has the value for the effects of local plate buckling and overall lateral torsional buckling.

In the first step the <u>elastic</u> lateral torsional buckling moment is calculated by eq. (7)

$$M_{E,ef} = M_E \sqrt{\frac{1}{1+\left(\frac{M_E}{M_{E,1}}\right)^2}} \tag{7}$$

where the following effects are considered:

- overall buckling by M_E
- local buckling by $M_{E,l}$
- overall and local buckling by $M_{E,ef}$

The elastic local buckling moment shall be determined from eq.(8).

$$M_{E,l} = k \cdot \sigma_e \cdot S \tag{8}$$

where

S elastic section modulus of the full unreduced section for the extreme compression fibre.

The procedure considering the plastic behaviour of the steel and initial imperfections corresponds to the design rules for compact sections. An equivalent slenderness is determined from eq.(9).

$$\bar{\lambda}_{M,ef} = \sqrt{M_{p,ef} / M_{E,ef}} \tag{9}$$

where the full plastic capacity of the effective section $M_{p,ef}$ is calculated taking into account the reduced cross section from fig. 3.

The reduction factor shall be determined as follows:

$$\kappa_M = \left(\frac{1}{1+(\bar{\lambda}_{M,ef})^{2n}}\right)^{1/n} \tag{10}$$

where

n = 2,0 for welded beams loaded by single loads
n = 1,6 for welded beams loaded by uniform bending.

Using the reduction factor κ_M the ultimate moment is calculated by eq. (11)

$$M'_u = M_u = \kappa_M \cdot M_{p,ef} \tag{11}$$

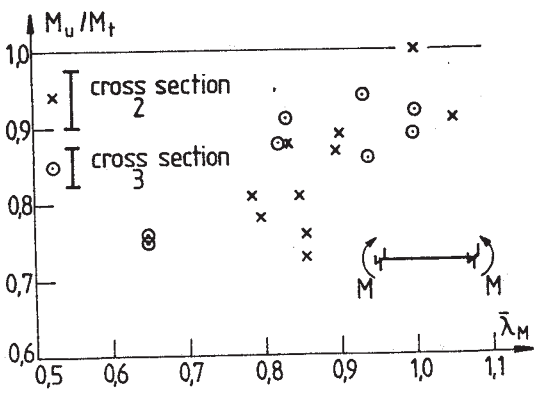

Fig. 3
Test results for design procedure A,
cross sections 2 and 3

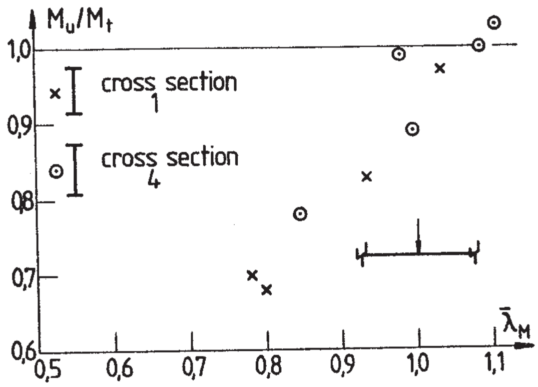

Fig. 4
Test results for design procedure A,
cross sections 1 and 4

According to this method the tests on cross sections 2 and 3 are evaluated. Cross section 2 was chosen considering that plate buckling of the web could occur. On the other hand as for cross section 3 plate buckling of the compression flange could occur. The results are given in fig. 3.

Fig. 3 shows, that this design procedure give conservative results. In all cases where overall lateral torsional buckling governs a good agreement between test results and design procedure can be recognized. In such cases where local plate buckling governs the design load is even more conservative.

The procedure described so far must be extended if the shear force exceeds a limiting value. Here a formulation is used which follows the DASt-Richtlinie 016 (Ref.11).

$$M/M_u' + V/V_u \leq 1{,}25 \qquad \text{for} \qquad 0{,}25 \leq \frac{V}{V_u} \leq 0{,}90 \tag{12}$$

The shear capacity is calcuted due to DIN 18800 part 3 (Ref. 10). Using

$$\bar{\lambda}_{P\tau} = \sqrt{\frac{f_y / \sqrt{3}}{\tau_{Pi}}} \tag{13}$$

$$\kappa_\tau = \frac{0{,}84}{\bar{\lambda}_{P\tau}} \tag{14}$$

$$V_u = A_w \, \kappa_\tau \, f_y / \sqrt{3} \tag{15}$$

$$V_t = A_w \, \tau_t \tag{16}$$

we get $M_u/M_t = 1{,}25/(M_t/M_u' + V_t/V_u)$ (17)

The results of all tests of cross sections 1 and 4 are shown in fig. 4.

In general there is a good correlation between test results and design load. The relationship M_u/M_t varies between 0,68 and 1,03 with a mean value of 0,87.

3.2 Design procedure B on the basis of a combined design curve

This design procedure is based on the new German stability code DIN 18800 part 3 (Ref.10) for plated structures. In (Ref.10) it is used for elements mainly subjected to axial forces. Here this procedure is extended to beams in bending which may fail by a combination of local plate buckling and overall lateral torsional buckling.

If only lateral torsional buckling shall be treated, eqs. (9) and (10) are valid but M_{el} instead of $M_{pl,ef}$ and M_E instead of $M_{E,ef}$ must be used. As a result the reduction factor κ_M is calculated.

If only local plate buckling must be treated further destinction is made between normal stresses σ, shear stresses τ and a combination of both stresses. For normal stresses σ eqs. (1) to (5) are valid if b_{ef}/b is replaced by the reduction factor κ_σ. For shear stresses the reduction factor κ_τ is calculated by eq. (14) using the relative slenderness $\bar{\lambda}_{p\tau}$ from eq (13).

If normal stresses σ as well as shear stresses τ are present a failure criterion is introduced, see eq. (17).

$$\left(\frac{\sigma}{\kappa_\sigma \cdot f_y}\right)^{e_1} + \left(\frac{\tau}{\kappa_\tau \cdot f_y/\sqrt{3}}\right)^{e_3} \leq 1 \qquad (17)$$

where

$$e_1 = 1 + \kappa_\sigma^4 , \qquad e_3 = 1 + \kappa_\sigma \, \kappa_\tau^2 \qquad (18)$$

For the evaluation of the test results the following procedure is used. Resulting from the tests the ultimate moment M_t and therefore the stresses σ and τ are known. A factor φ is calculated iteratively in such a way that by multiplying σ and τ by φ eq. (17) is fulfilled. Therefore the reduction factor κ_P for a combined stress level of σ and τ is given by eq. (19)

$$\kappa_P = \varphi \, M_t/M_{el} \qquad (19)$$

In order to take into account the mutual influence of overall lateral torsional buckling and local plate buckling eq. (20) is accounted for.

$$\kappa_{MP} = \kappa_M \, \kappa_P \qquad (20)$$

The ultimate moment M_u then is calculated by eq (21).

$$M_u = \kappa_{MP} \, M_{el} \qquad (21)$$

The results of the evaluation of all test results can be seen in figs. 5 and 6.

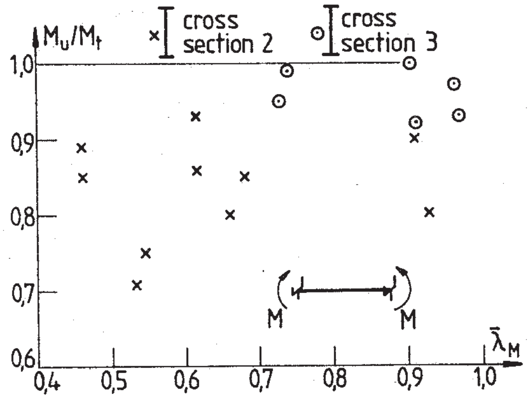

Fig. 5
Test results for design procedure B,
cross sections 2 and 3

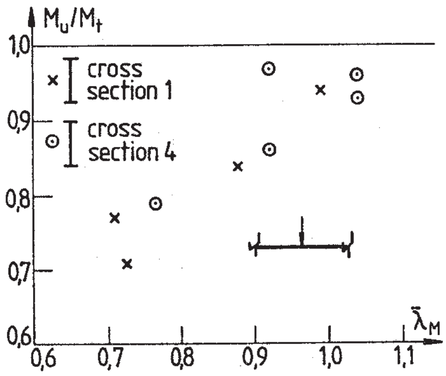

Fig. 6
Test results for design procesure B,
cross sections 1 and 4

These figs. show that for all tests the design values are conservative. But in general there is a very good agreement. The realtionship between design value and test result varies between 0.71 and 1.0. Especially in tests 13 to 18 where local plate buckling of the flange dominates this relationship varies between 0.87 and 1.0 only.

4. CONCLUSION

Two design procedures are explained. They are based on the new German stability code DIN 18800 part 2 and 3. Comparisons with test results show a good correlation.

5. ACKNOWLEDGEMENTS

The author would like to express his appreciation to the "Deutsche Forschungsgemeinschaft" (DFG) for providing financial assistance and to Dipl.-Ing. T. Gregull/Fa. Krupp Stahlbau Berlin for conducting the tests.

REFERENCES

(1) Scheer, J.: Zum Problem der Gesamtstabilität von einfachsymmetrischen I-Trägern. Stahlbau 28 (1959), S. 113.

(2) Fischer, M.: Das Kipp-Problem querbelasteter exzentrisch durch Normalkraft beanspruchter I-Träger bei Verzicht auf die Voraussetzung der Querschnittstreue. Stahlbau 36 (1967), S. 77.

(3) Sedlacek, G. und Stoverink, H.: Zum Biegedrillknicknachweis von Hallenrahmen mit voutenförmig ausgebildeten Stützen und Riegeln. Stahlbau 55 (1986), H. 8, S. 225-232.

(4) Fukumoto, Y.; Maegawa, K. und Itoh, Y.: Lateral-Torsional Buckling Strength of Welded I-Girders. Stahlbau 55 (1986), H. 8, S. 237-245.

(5) Lindner, J. und Gregull, T.: Design Rules for Thinwalled Structures according to DIN 18800 Part 2. International Conference on Steel and Aluminium Structures, Cardiff, Juli 1987.

(6) Fukumoto, Y. und Kubo, M.: Lateral torsional buckling of thin-walled I-beams. Journal of Structural Engineering, Vol. 114, No. 4, S. 841-855.

(7) Lindner, J. : Developments on lateral torsional buckling. 2nd International Colloquium on Stability, Washinton D.C., May 1977, pp.

(8) Specification for the Design of Cold-formed Steel Structural Members, AISI Report 5686-2, Washington D.C., 1986.

(9) DIN 18800 Teil 2. Stahlbauten - Stabilitätsfälle, Knicken von Stäben und Stabwerken. (Steel structures, stabilits, buckling of bears and skeletal structures), (11.90).

(10) EDIN 18800 Teil 3: Stahlbauten - Stabilitätsfälle, Plattenbeulen. (Steel structures, stability, buckling of plates), (11.90).

(11) DASt-Richtlinie 016: Bemessung und konstruktive Gestaltung von Tragwerken aus dünnwandigen, kaltgeformten Bauteilen. (Design of thinwalled coldformed structures). Köln. Stahlbau-Verlag, 1988.

(12) Lindner, J.; Gregull, Th.: plastische Interatkion (plastic interaction), Schlußbericht zum DFG-Forschungsvorhaben Li 351/4-1. Institut für Baukonstruktionen und Festigkeit TU Berlin, 1990.