RESTRAINT OF BEAMS BY TRAPEZOIDALLY SHEETING
USING DIFFERENT TYPES OF CONNECTION

Univ.-Prof. Dr.-Ing. J. Lindner
Technische Universität Berlin
Fachgebiet Stahlbau
Hardenbergstr. 40A, 12623 Berlin, Germany

ABSTRACT

The stability problem of lateral torsional buckling is substantially reduced by adjacent members like sheeting where stabilization by shear stiffness and by torsional restraint is present. Here the torsional restraint is investigated which is especially influenced by the type of connection. Tests were carried out for the connection by shot fired pins and under high loads. The results are given as values for the torsional restraint coefficient $c_\theta$ of the actual configuration. Account is taken of the width $b_0$ of the beam flange, the thickness $t$ of the sheeting, the type of loading and the magnitude of the load $A$ introduced from sheeting to beam.

Two examples illustrate the application.

KEYWORDS
beams, stability, lateral torsional buckling, stabilization, torsional restraint coefficient, tests, connection, shot fired pins.

1. INTRODUCTION

Slender beams may buckle by a combination of lateral bending and twisting. This stability problem of lateral torsional buckling is especially of interest for beams like purlins. For an easy treatment the beam is usually assumed as an unsupported beam with simple supports. Most specifications define rules for unsupported beams.

In reality unsupported beams as assumed in codes are very rare in practical design. All loads are introduced by neighbouring separate members and their stiffness can be taken into account in much situations. Therefore the risk of lateral torsional buckling may be substantially reduced by adjacent members.

2. INFLUENCE OF RESTRAINT BY ADJACENT MEMBERS

Adjacent members are normally present as individual members like cross beams or as floor elements like sheeting. The restraining effect of cross beams is mainly caused by its bending stiffness and to some extent
by the joints. Contrary to this for sheeting commonly two restraining effects are present:

i) the horizontal deflection of the upper cord of the beam is prevented by the shear stiffness $R$ of the sheeting, if fasteners between sheeting and beam are present,

ii) the bending resistance of the sheeting in combination with local deformations partly prevents the twisting of the beam.

The effect of horizontal restraint is remarkable, Heil (1994), Lindner (1996) and is in some cases sufficient to reach satisfactory capacity. In other cases additionally the effect of torsional restraint is needed. Therefore here this effect is dealt with only.

It is assumed as a basis for the following design procedure that the torsional restraint acts as an elastic restraint. Therefore it is assumed that the trapezoidally sheeting does not reach its ultimate capacity, which is followed by plastic deformations, at the same time when the beam tries to twist.

A simplified stability check requires a minimum stiffness of the torsional restraint coefficient $c_0$ by Eqn. 1.

$$c_0 \geq \left( \frac{M_{pl}^2}{EI_x} \right) k_0$$

(1)

<table>
<thead>
<tr>
<th>Top flange</th>
<th>Moment distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free</td>
<td>$M$</td>
</tr>
<tr>
<td></td>
<td>4.0</td>
</tr>
<tr>
<td>Restraint</td>
<td>0</td>
</tr>
</tbody>
</table>

The factor $k_0$ depends on the moment distribution and the fact whether the upper flange of the beam is horizontally restrained or not. Design values for $k_0$ can be seen from Table 1, see DIN 18 800 part 2.

If the simplified stability check of Eqn. 1 is not used, the positive influence of the torsional restraint from trapezoidally sheeting can be accounted for by calculating the elastic critical moment $M_{cr}$, taking into account $c_0$ directly.

The effective torsional restraint coefficient $c_0$ depends on the rigidity of the joint between the beam which should be stabilized and the structural element which acts as an adjacent member. For trapezoidally sheeting three deformation components should be taken into account as shown by Eqn. 2.

$$\frac{1}{c_0} = \frac{1}{c_{0,M}} + \frac{1}{c_{0,A}} + \frac{1}{c_{0,P}} \quad [kN/m]$$

(2)

where

$c_{0M}$ theoretical value with regard to the stiffness of the adjacent member only

$= 4 E I_y / L_s$ for continuous sheeting,

$I_p I_s$ effective moment of inertia, span of the adjacent member,
c_{sp} \quad \text{distorsion of the beam investigated}
\quad = 5770(d/lw^3 + 0.5 b/yw^2), \text{dimensions in [cm]}

\text{c}_{\text{ba}} \quad \text{rigidity of connection.}

The rigidity of connection \(c_{\text{ba}}\) is most important especially for trapezoidally sheeting. Values for \(c_{\text{ba}}\) can be obtained by tests only.

Important parameters which influence the results significantly are (see Lindner et al (1986), Lindner et al (1989), Lindner et al (1996)):

- type of the roofing skin (e.g. depth and width of the trapezoidally sheeting, plate thickness \(t\)),
- positioning of sheeting (positive or negative),
- location and distance of the fasteners (trough, crest, fasteners in every \((b=sh)\) or alternate \((e=2\times h)\) trough/crest ),
- type of fasteners (self-tapping screws diameter 6.3 mm, shot fired pins),
- roofing construction (with or without intervening thermal insulation, type of thermal insulation),
- type of loading (gravity load, uplift load by wind),
- magnitude of loading,
- width \(b\) of the beam.

Values of \(c_{\text{ba}}\) given earlier are based on tests with a plate thickness of \(t = 0.75\ \text{mm}\), a magnitude of the load acting on the beam of \(1.0\ \text{kN/m}\) and self-tapping screws. They are included in relevant codes in Europe, like DIN 18 800 part 2 and Eurocode 3 part 1.3.

3. TEST CONFIGURATION

It was shown that a segment of a realistic constructed roof section may be used in tests for different types of loading and constructional details. Test configuration and conducting of the tests were described earlier, (see Lindner et al (1986), Lindner et al (1989), Lindner et al (1996)), see Figure 1.

![Test setup diagram](image)

Figure 1: Test set up

Three different types of beams were used in the tests: IPE 160, HE 160A and IPE 160+U200, see Figure 2. These profiles have a width \(b\) of the flange of 82, 160, 200 mm. The last combined profile was used because no other profile of \(b = 200\ \text{mm}\) was available.

As trapezoidally sheeting the types of E40/183 with \(t = 0.75\ \text{and} 1.00\ \text{mm}\) and E85/280 with \(t = 0.75\ \text{mm}\) where used in positive and negative position, see Figure 3.
Figure 2: Investigated purlins

Figure 3: Investigated trapezoidally sheeting

The connection between purlin and sheeting was executed by selftapping screws with a diameter of 6.3 mm or shot fired pins of HILTI type ENP2-21L15 which have an approval by Deutsches Institut für Bautechnik. The connectors were placed in every bottom chord ($e = b_r$) or in every second bottom chord of the trapezoidally sheeting ($e = 2b_r$), see Figure 4.

Figure 4: Example for configurations investigated in tests

Different test series were carried out in the years 1994-1996 to investigate especially the influence of the magnitudes of the load for gravity load, the width $b_r$ of the flange and uplift loading. The results are an extension of the rules in codes already used. They are summarized in the following.

4. TEST CONDUCTION AND TEST RESULTS

The torque $m$, to the beams is introduced by alternating loads at the cantilevers, see Figure 1, such leading to hysteresis loops. From deflections at different locations of the purlins the twisting $\Theta$ is calculated.
An example for a torque moment-twist curve measured during the tests is shown in Figure 5. From this figure it can be seen that the stiffness related to a small value \( \Theta \) can be much higher than for the value \( \Theta = 0.1 \) which was taken as an unfavourable reference value. All design values given in DIN 18 800 part 2, Eurocode 3 and Lindner et al (1996) are based on this unfavourable assumption of \( \Theta = 0.1 \).

![Figure 5: Example for a torque moment-twist curve from test](image)

From the point of practical application it seems to be suitable to use the results in the same way as proposed in the stability code DIN 18 800 part 2 and Eurocode 3 part 1.3. Therefore a basic connection stiffness \( \widetilde{c}_{BA} \) related to a flange width of \( b_f = 100 \text{ mm} \) is defined and given in Table 2. All other parameters are taken into account by additional factors with regard to Eqn. 3.

\[
c_{BA, \Theta} = \widetilde{c}_{BA, \Theta} \cdot k_a \cdot k_f \cdot k_r \quad (3)
\]

where

\[
k_a = \left( \frac{b_f}{100} \right)^3 \quad \text{if} \quad b_f \leq 125 \text{ mm} \quad (4a)
\]

\[
k_a = \left( \frac{b_f}{100} \right) 1.25 \quad \text{if} \quad 125 < b_f \leq 200 \text{ mm} \quad (4b)
\]

\[
k_r = \left( \frac{t}{0.75} \right)^{1.1} \quad \text{positiv position} \quad (5a)
\]

\[
k_r = \left( \frac{t}{0.75} \right)^{1.5} \quad \text{negativ position} \quad (5b)
\]

for gravity load:

\[
k_A = 1.0 + (A - 1.0) \cdot 0.08 \quad \text{if} \quad t = 0.75 \text{ mm} \quad \text{positive position} \quad (6a)
\]

\[
k_A = 1.0 + (A - 1.0) \cdot 0.16 \quad \text{if} \quad t = 0.75 \text{ mm} \quad \text{negative position} \quad (6b)
\]

\[
k_A = 1.0 + (A - 1.0) \cdot 0.095 \quad \text{if} \quad t = 1.00 \text{ mm} \quad \text{positive position} \quad (6c)
\]

\[
k_A = 1.0 + (A - 1.0) \cdot 0.095 \quad \text{if} \quad t = 1.00 \text{ mm} \quad \text{negative position} \quad (6d)
\]

for uplift load:

\[
k_A = 1.0 \quad (6c)
\]

\[\Lambda \leq 12 \text{ [kN/m]} \text{ load introduced from sheeting to beam}\]

\[t \text{ [mm]}, b_f \text{ [mm]}\]

It can be seen from Table 2 that for selftapping screws and the special type of shot fired pins investigated here the same values may be used with the extension of one case.
### TABLE 2
CHARACTERISTIC VALUES FOR THE CONNECTION STIFFNESS $\varepsilon_{\text{act}}$, VALID FOR BEAMS WIDTH $b_r = 100$ mm, FASTENING AT BOTTOM CHORD OF TRAPEZOIDALLY SHEETING

<table>
<thead>
<tr>
<th></th>
<th>position of sheeting</th>
<th>distance of fasteners</th>
<th>selfflapping</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>positive</td>
<td>negative</td>
<td>$e / b_r$</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>gravity load:</td>
<td>x</td>
<td>x</td>
<td>5.2</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>2.0</td>
</tr>
<tr>
<td>uplift load:</td>
<td>x</td>
<td>x</td>
<td>2.6</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td>x</td>
<td>x</td>
<td>0.6</td>
</tr>
</tbody>
</table>

#### 5. STATISTICAL EVALUATION OF THE TESTS

In order to find out whether the proposed characteristic values of Eqn. 3 in combination with Table 2 have a sufficient safety level statistical evaluations were carried out.

For a group of 111 tests for gravity load the relation $\alpha$ between test result and design value calculated by Eqn. 3 is calculated. The results can be seen from Table 3.

#### TABLE 3
TEST EVALUATION FOR THE RELATION FACTOR $\alpha$ FOR GRAVITY LOAD

<table>
<thead>
<tr>
<th></th>
<th>m</th>
<th>1.436</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean value</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>standard deviation</td>
<td>m / m</td>
<td>0.222</td>
</tr>
<tr>
<td>standard dev. /mean value</td>
<td>s / m</td>
<td>0.159</td>
</tr>
<tr>
<td>lower limit</td>
<td>m - 0.8s</td>
<td>0.992</td>
</tr>
<tr>
<td>simplified lower limit</td>
<td>0.8m</td>
<td>1.149</td>
</tr>
</tbody>
</table>

Similar results were obtained for 36 test for uplift load. Because of a higher scattering of the results it was necessary to use a statistical factor of 0.7 instead of 0.8 to get sufficient results.

#### 6. EXAMPLES

##### 6.1 Purlin

A roof is investigated with purlins of IPE 240 section as continuous beams, span $L = 11$ m, loaded by uniformly distributed load. The trapezoidally sheeting 40x183x0.75 mm is fastened at the bottom chord, $e = b_r$. 
It can be shown due to the rules of DIN 18 800 part 2 that the shear stiffness of the sheeting leads to a horizontal restraint of the upper chord.

\[
\begin{align*}
M_{px,t} & = 88 \text{ kNm} \\
I_s & = 284 \text{ cm}^4 \\
c_{bM} & = 4 \cdot 21000 \cdot 0.00216 / 3.0 = 60.5 \text{ kNm/m} \\
c_{bP} & = 5770 / (24 / 0.62^3 + 0.5 \cdot 12 / 0.98^3) = 53.9 \text{ kNm/m} \\
\text{from Table 2:} \\
\overline{c}_{\theta,A,,t} & = 3.1 \text{ kNm/m} \\
\text{Eqn. 4b:} \\
c_{b,A,t} & = 3.1 \times 120 / 100^2 = 4.46 \text{ kNm/m} \\
\text{Eqn. 3:} \\
1/c_\theta & = 1/60.5 + 1/53.9 + 1/4.46 \\
c_\theta & = 3.86 \text{ kNm/m} \\
\text{Eqn. 1 and Table 1:} \\
c_\theta & \geq (88.0^2 / (21000 \cdot 0.0284)) \times 0.23 = 2.99 \text{ kNm/m} < 3.86 \text{ kNm/m}
\end{align*}
\]

Because the minimum stiffness requirement under the simplified assumption of \(k_A = 1.0\) is fulfilled no further check for the parlin with regard to lateral torsional buckling is necessary.

6.2 **Rafter of frame**

The rafter of a frame with the span length of \(L = 15\) m, the profil IPE 400 and steel grade St 37 (Fe 360) is investigated as an individual member, see Figure 7. The actions are given by

- **permanent action**
  
  \[ g = 0.95 \text{ kN/m}^3, \]

- **variable action**
  
  \[ p = 0.75 \text{ kN/m}^3, \]

- **total load**
  
  \[ q = 1.70 \text{ kN/m}^3. \]

\[
\begin{align*}
M_{f,k} \text{ [kNm]} \\
\end{align*}
\]

Figure 7: Moment distribution
The rafter is restraint by trapezoidal sheeting E100/t=1.00 mm in positive position, connected every 2. trough (ε = 2h). The sheeting runs from rafter to rafter as a two span member with a = 6.0 m span length.

Using the partial safety factors from DIN 18 800 including γ_m = 1.1

\[ A = 1.25 \cdot 1.1 \cdot (1.35 \cdot 0.95 + 1.50 \cdot 0.75) \cdot 6.0 = 19.9 > 12.0 \text{ kN/m} \]

The shear stiffness is calculated due to the approval ("Typenprüfung") of Hoesch (1989).

\[ K_s = 0.188 \text{ m/kN} \]
\[ K_s = 16.6 \text{ m}^2/\text{kN} \]
\[ L_s = 2 \cdot 6.0 = 12.0 \text{ m} \]
\[ S = 10^4 / (0.188 + 16.6 / 12) = 6364 \text{ kN} \]
\[ S_s = 6364 \cdot 6.0 = 38180 \text{ kN} \]

The shear panel is connected to the rafters and to the edge members as well. But the sheeting is connected in every second rib only and therefore due to DIN 18 800 part 2 20% of S_s can be used only.

\[ R = \frac{S_s}{5} = 7640 \text{ kN} \]

In order to check whether full lateraly restraint of the top chord may be assumed Eqn. 7 of DIN 18 800 part 2 is used

\[ \lim R = \left( E I_o \frac{\pi^2}{L^2} + G I_T + B I_s \frac{\pi^2}{L^2} 0.25 h^2 \right) \frac{70}{h^2} \]

where

\[ I_o \text{ warping constant} \]
\[ I_T \text{ S.Venant torsion constant} \]
\[ I_s \text{ moment of inertia weak axis} \]

\[ \lim R = (21000 \cdot 0.0049 \pi^2 / 15^2 + 8100 \cdot 0.00514 + 21000 \cdot 0.132 \pi^2 0.25 \cdot 0.4^2 / 15^2 ) 70 / 0.4^2 \]
\[ = 22300 \text{ kN} \]

No full restraint can be assumed because of \( R < \lim R \).

Simplified check by Eqn. 1:

\[ \text{req.} c_{b,\varepsilon} = \left( 314^2 / (21000 \cdot 0.132) \right) 3.5 = 124 \text{ kNm/m} \]
\[ b_l / 100 = 180 / 100 = 1.80 \]
\[ k_o = 1.25 \cdot 1.8 = 2.25 \]
\[ k_s = (1.0 / 0.75)^{1.1} = 1.37 \]
\[ k_A = 1.0 + (12.0 - 1.0) \cdot 0.095 = 2.05 \]
\[ c_{b,\varepsilon} = 3.1 \text{ kNm/m} \]
Eqn. 3: \[ c_{BA,3} = 3.1 \cdot 2.25 \cdot 1.37 \cdot 2.05 = 19.6 \text{ kNm/m} \]
\[ c_{EF} = 120 \text{ kNm/m} \]
\[ c_{RM} = 137 \text{ kNm/m} \]
\[ c_r = \frac{1}{(1/120 + 1/137 + 1/19.6)} = 15.0 \text{ kNm/m} \]

The first two values for \( c_{EF} \) and \( c_{RM} \) are taken from the book Lindner et al (1994).

The simplified design check leads to:

15.0 < 124 kNm/m

and is therefore not sufficient.

Therefore another design check is necessary. In doing this the positive action of the stiffnesses \( c_r \) and \( R \) are accounted for by calculating the elastic critical moment \( M_{cr} \), see Bamm et al (1995).

The results are given in Table 4.

<table>
<thead>
<tr>
<th>R</th>
<th>( c_r )</th>
<th>( M_{cr} )</th>
<th>( x_M )</th>
<th>( M_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[kN]</td>
<td>[kNm/m]</td>
<td>[kNm]</td>
<td>DIN 18 800 part 2</td>
<td>DIN 18 800 part 2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>68.5</td>
<td>0.216</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>15.0</td>
<td>233</td>
<td>0.638</td>
</tr>
<tr>
<td>3</td>
<td>7640</td>
<td>0</td>
<td>381</td>
<td>0.826</td>
</tr>
<tr>
<td>4</td>
<td>7640</td>
<td>15.0</td>
<td>1105</td>
<td>0.983</td>
</tr>
</tbody>
</table>

It can be seen that the influences of the torsional restraint coefficient \( c_r \) and the shear stiffness \( R \) are effective and similar in the result. If both stiffnesses are taken into account together 98.3 % of the full plastic moment is reached and therefore the design check leads to 309 > 303 kNm.

7. CONCLUSIONS

Beams which are prone to lateral torsional buckling are restraint in many cases by adjacent members. It is reported on new investigations concerning the torsional restraint by trapezoidally sheeting. For the first time the effects of shot fired pins and the magnitude of the load were accounted for. The results are given as torsional restraint coefficients which can directly be used for practical design. Regulations already given in DIN 18 800 part 2 and Eurocode part 1.3 can be extended by these proposal. The positive effect of the restraint by trapezoidally sheeting are demonstrated by two examples.

Parts of these investigations were sponsored by HILTI/Liechtenstein. At the Technical University of Berlin Mrs. Grams, Mrs. Dubsky, Mr. Jacobs and Mr. Hermann conducted most of the tests which was gratefully acknowledged.
References


Lindner, J. und Groeschel, F.: Drehbettungswerte für die Profilblechbefestigung mit Setzbolzen bei unterschiedlich großen Auflasten (Torsional restraint coefficients for connections with shot fired pins and variable loads), Stahlbau 65(1996), S. 218-224.