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# Consideration of initial imperfections for members subjected to axial compression and bending

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ABSTRACT: For the analysis of a structure the initial imperfections must be chosen in such a way that the most unfavourable influence on the ultimate capacity is taken into consideration. The choice of the form of the initial imperfection is easy in usual cases: for a member with simple supports subjected to axial forces the form must be similar to a sine wave.

For other types of structures, especially continuous beams which are loaded by axial forces and bending moments the choice of the shape of imperfection is not as easy: for a two-span beam loaded by axial forces and transverse loads it is imaginable that an initial imperfection similar to the deflection caused by the transverse loads leads to a smaller ultimate load carrying capacity than an imperfection according to the first eigenmode. Parameter studies were carried out for two-span and three-span beams. For different shapes of imperfections the relation between the magnitude of the axial force and the bending moments was varied as well as the type of cross section and the axis of buckling: weak axis buckling and strong axis buckling. Furthermore different methods of analysis were dealt with, such as ultimate load calculations taking into account the yielding along the beam, plastic hinge theory and elastic second order theory. The results are shown in diagrams.

## 1 INTRODUCTION

### 1.1 *General*

Most members in steel structures are beam columns and therefore subjected to combined action of compression and bending. The borderline cases are members loaded by axial compression only (columns) and members loaded by pure bending (beams). However, the common case is a combined load and because of the great importance the behaviour and design of beam columns has been a subject of research for the last 70 years.

The intention was and still is to find out the ultimate limit state of a structure taking into account a correct material law, residual stresses, unavoidable geometrical imperfections and boundary conditions. In an exact investigation all of this points have to be taken into account. Depending on the complexity of such calculations usually different simplifications are made which lead to different levels of accuracy for the results. In general two ways to treat a structure are common.

On the one hand, the investigated beam column is cut off from the overall structure and the interaction with the other members has to be taken into account with special boundary conditions. This can be done easily using an appropriate buckling length. The stability check is usually done by using an interaction formulae on the basis of a buckling curve (e.g. European Buckling Curves). This buckling curves regards the influences of geometrical and structural imperfections dependent on the shape of the cross section of the member. Those interaction formulae are simple to handle and thus part of various design codes.

On the other hand, due to the development of large capacity computers, it is more and more common to investigate a structure as a whole using the second order theory. Therefore, the influence of geometrical and structural imperfections has to be taken into account in a convenient manner. This can be done by using equivalent geometrical imperfections or equivalent loads (see Fig. 1). The maximal size $w_0$ of the initial imperfection depends on the geometry of the cross section and its assignment to a buckling curve. Examples for the European buckling curves are given in Table 1 (see (DIN 18800-2 1990)). It has to be noted, that the initial deflections in fact depend on the

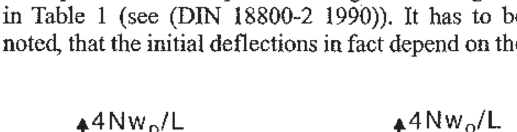


Figure 1. Equivalent imperfection loads (due to quadratic initial bow imperfection) according to DIN 18800-2.

Table 1. Geometrical equivalent imperfections.

| Cross section with regard to European buckling curve | $v_0, w_0$ |
|---|---|
| a | L/300 |
| b | L/250 |
| c | L/200 |
| d | L/150 |

nondimensional slenderness and the steel grade. The denoted values in Table 1 are a lower limit and for that on the save side in most cases (see Lindner *et al.* 1998). Using the internal forces a check of the cross section capacity based on the theory of elastic or plastic design covers the stability check. Depending on the definition of the global ultimate limit state the load carrying capacity of the structure is reached if the yield stress is reached in the most unfavourable fibre, a first plastic hinge (elastic–plastic) occurs or the last plastic hinge of the structure leads to a failure mechanism.

The second way is more complex but on the other hand gives more exact results. The initial imperfections must be chosen in such a way that the most unfavourable influence on the ultimate load carrying capacity of an investigated system is taken into consideration. The choice of the form of the imperfection is easy in usual cases: for a single span member with simple supports subjected to axial forces the form must be similar to the first sine wave and is therefore symmetrical with regard to the middle of the member.

For other types of structures, especially continuous beams which are loaded by axial forces and bending moments the choice of the type of imperfection is not as easy: for a two-span beam loaded by axial forces only an asymmetrical imperfection is the most unfavourable one. In the first span the deformation must be downwards, in the second span upwards or vice versa (see Fig. 2/I).

But contrary to that for a loading mainly caused by bending moments in both spans the deflection is, due to the loads, downwards or upwards in both spans (see Fig. 2/III). If there is a combination of both loadings, axial forces and bending moments, it is not clear how the initial imperfection should be chosen. It can be suspected that this depends on the relation between the two types of loading.

This problem was discussed by some authors only, e.g. (Lehmkuhl 1999), (Rubin 2000), (Nather 2001) and (Glitsch 2002), but no final conclusions could be drawn.

### 1.2 *Notation*

The following symbols are used in this paper:

$A$ area of cross section
$I_y$ moment of inertia due to the strong axis y-y

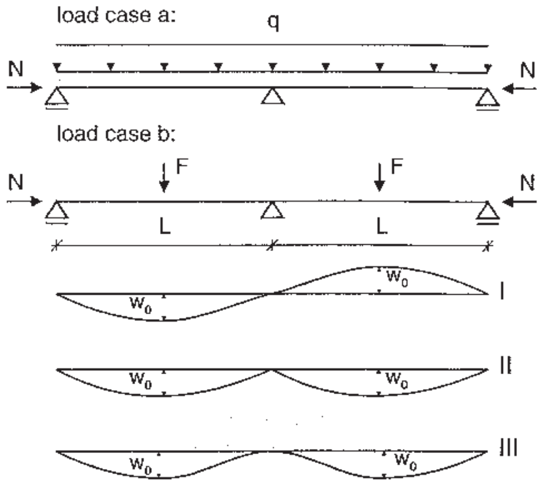


Figure 2. Two-span beam, load cases and imperfections.

$I_z$ moment of inertia due to the weak axis z-z
$L$ length of the member
$h, b$ overall depth, width of the cross section
$f_y$ yield strength
$N_{pl}$ $= Af_y$, full plastic axial force
$M_{pl}$ $= W_{pl}f_y$, full plastic moment
$M_{pl,N}$ reduced full plastic moment due to an axial force $N$
$N_{cr}$ elastic critical load for flexural buckling
$N_u$ ultimate axial force
$\kappa$ $= N_u/N_{pl}$, reduction factor
$\bar{\lambda}_y, \bar{\lambda}_z$ nondimensional slenderness

## 2 THEORETICAL BACKGROUND

### 2.1 *General*

For the following investigations the second order plastic hinge analysis (SOPHA) and the geometrical and material nonlinear finite element method (FEM) as a more exact analysis are used.

Two systems, a two-span beam and a three-span beam, with equal span lengths are investigated according to this two methods considering different shapes of imperfection. In order to get a good survey of the behaviour of the beam columns, the lengths (which leads to different nondimensional slendernesses $\bar{\lambda}_y, \bar{\lambda}_z$) and the ratio between axial loads N and transverse loads $q$ respectively $F$ (see Fig. 2) are varied.

### 2.2 *Assumptions*

If not otherwise mentioned the following assumptions are made:

– in-plane flexural buckling only is taken into account,

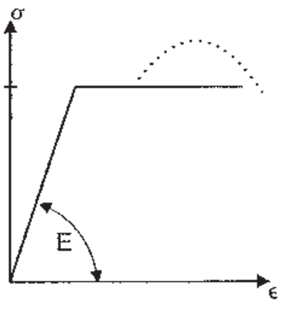


Figure 3. $\sigma$–$\varepsilon$-relationship.

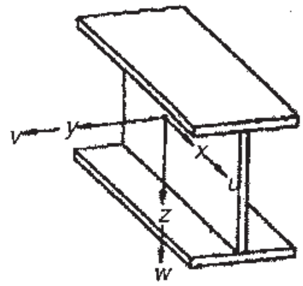


Figure 4. Coordinate plan.

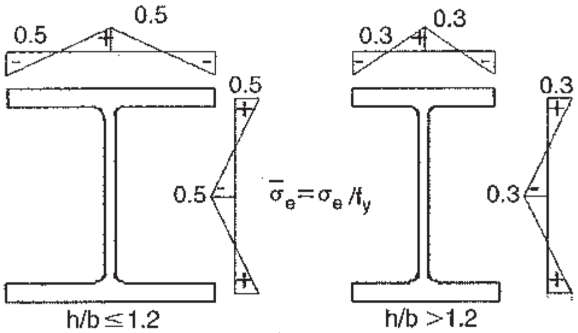


Figure 5. Residual stresses for I-beams, linear shape.

- all sections are compact and doubly symmetrical,
- the cross section is constant along the member,
- the material behaviour is defined with a bilinear elastic-perfectly plastic stress-strain relationship without hardening effects (see Fig. 3),
- the distribution of the residual stresses follows Fig. 5, see (ECCS-CECM-EKS 1984),
- the form of the geometrical imperfections is a sine wave or parabolic with a maximum size of $v_0 = L/1000$ or $w_0 = L/1000$ (see Fig. 2),
- the influence of shear stresses with respect to the full plastic moment is neglected.

The orientation of cross section to the coordinate plane is shown in Fig. 4.

Table 2. Section properties and initial equivalent imperfections.

| | | IPE 200 | HEB 300 |
|---|---|---|---|
| $A$ | [cm$^2$] | 27.25 | 142.8 |
| $I_y$ | [cm$^4$] | 1846.0 | 24190.0 |
| $I_z$ | [cm$^4$] | 141.9 | 8553.0 |
| $N_{pl}$ | [kN] | 640.3 | 3356.0 |
| $M_{pl,y}$ | [kNm] | 49.27 | 420.8 |
| $M_{pl,z}$ | [kNm] | 10.34 | 202.9 |
| $w_0$ | [cm] | $L/300$ | $L/250$ |
| $v_0$ | [cm] | $L/250$ | $L/200$ |

### 2.3 *Cross sections*

All investigations were made with typical hot-rolled double symmetrical cross sections: IPE 200 with $h/b > 1.2$ and a wide flange section HEB 300 with $h/b \leq 1.2$. With regard to the european buckling curves these sections belong to different curves, which can also be mentioned from Table 2 because of different imperfections. The radius is neglected. The most important section properties are given in Table 2. The yield strength is defined with 235 Mpa ($= 235\,\text{N/mm}^2$) which corresponds to the typical mild steel grade of S 235.

## 3 RESULTS

### 3.1 *Two-span beam*

For a two-span beam subjected to axial forces and distributed load (load case a) respectively point loads at midspan (load case b) the influence of different assumptions for the initial imperfections are investigated (see Fig. 2). Initial deflection type *I* corresponds to the bifurcation mode for axial forces $N$, initial deflection type *III* corresponds to the deflection due to the transverse loads.

There is no difference made for shape *III* between the distributed load and the point load because of the small influence and the better handling. The initial imperfections are taken into account using equivalent loads for the second order plastic hinge analysis. For the FEM analysis the imperfections are included correctly (see Fig. 1). Shape *II* is an alternative to shape *III* and used if imperfections are directly considered without using equivalent imperfection loads (e.g. in computer programs).

An exact analytical solution for the internal forces according to elastic second order theory can be found in (Vogel & Rubin 1982) and (Nather 2001). These results can be used for second order plastic hinge theory too. The effect of axial forces on the capacity of the full plastic moment, called $M_{pl,N}$, is reported by (Lindner 1993) and (Chen & Sohal 1995). The

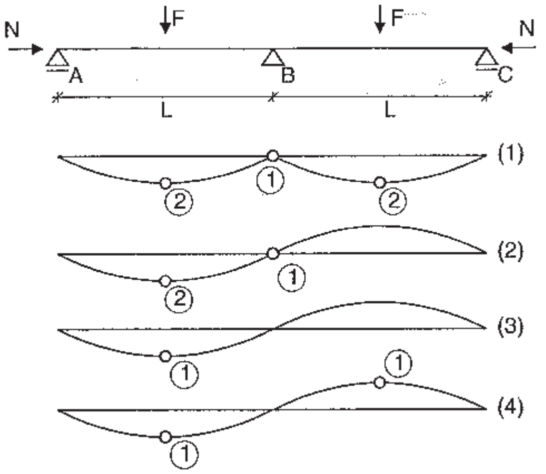


Figure 6. Failure mechanism.

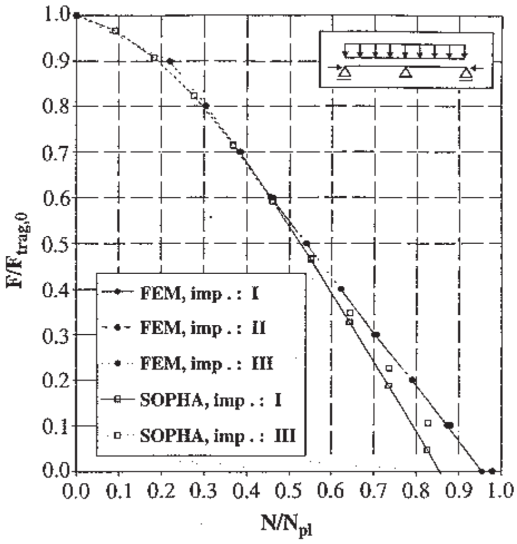


Figure 7. IPE 200, $N + M_y$, $\bar{\lambda}_y = 0.5$, LC a.

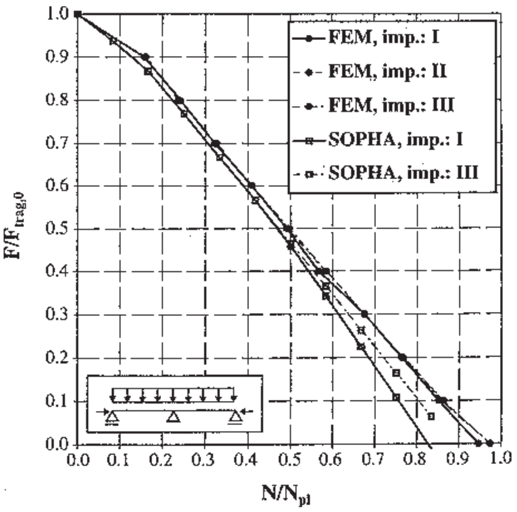


Figure 8. HEB 300, $N + M_y$, $\bar{\lambda}_y = 0.5$, LC a.

equivalent imperfection loads are calculated by Eq. 1 and Eq. 2.

$$F_e = N \cdot 4\frac{w_0}{L} \tag{1}$$

$$q_e = N \cdot 8\frac{w_0}{L^2} \tag{2}$$

Depending on the chosen initial imperfection and the ratio between axial forces and bending moments different failure mechanisms occur. For the load case point loads these mechanisms are shown in Fig. 6.

In case (1) a symmetrical initial imperfection leads to a first plastic hinge at support B. Next, there will occur plastic hinges at both midspans simultaneously. Therefore the ultimate load of the system is reached by a kinematic mechanism. This behaviour is independent of the ratio between axial load and transverse load because initial imperfection and load deflection have the same direction.

Very different from that is the behaviour if an unsymmetrical initial imperfection is assumed. Here the failure mechanism depends directly on the ratio between axial load and transverse load. If there is only a small axial load, case (2) occurs. The first plastic hinge is again located at the support B but after that the second plastic hinge will appear at midspan of the span where initial imperfection and load deflection have the same sign. Increasing of the load is impossible although one span is still statically determined.

If the axial load increases with respect to the point loads case (3) and at last case (4) will occur. In these cases the system changed significantly. In case (3) the first plastic hinge occurs now at midspan and therefore the ultimate load is less than the ultimate load of a continuous beam. Here the ultimate limit state is reached with this first plastic hinge.

Case (4) occurs if the axial load increases compared to the point loads.

For the presentation of the results the following procedure is chosen:

1. calculation of the ultimate transverse load $F_{trag,0}$ according to $N = 0$ (first order ultimate limit state)
2. calculation of the ultimate axial force $N_u$ according to a ratio $\eta = F/F_{trag,0}$ for $\eta = 0.1, 0.2, \ldots 1.0$

The FEM results were calculated using the program system ANSYS 5.7.1 taking into account a material and geometrical nonlinear BEAM element (*BEAM188*, see (ANSYS, Inc. 2000)), whereby the

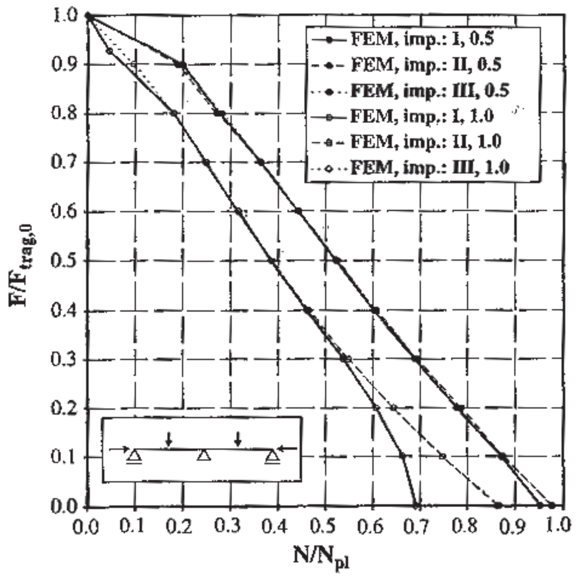


Figure 9. IPE 200, $N + My, \overline{\lambda}_y = 0.5$ and 1.0, LC b.

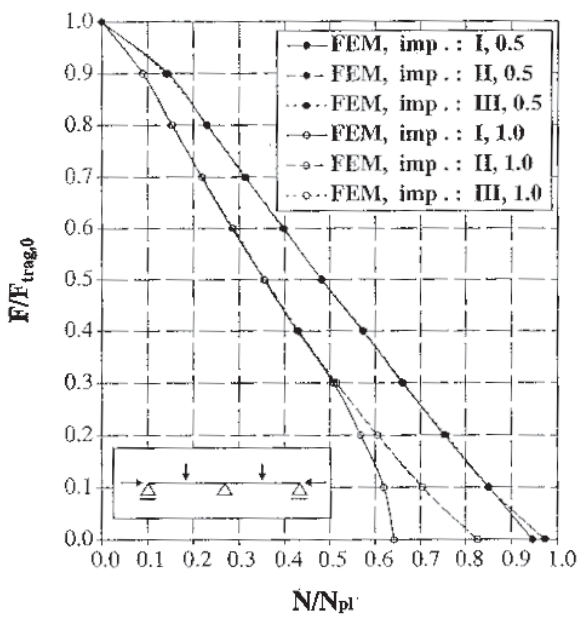


Figure 10. HEB 300, $N + M_y, \overline{\lambda}_y = 0.5$ and 1.0, LC b.

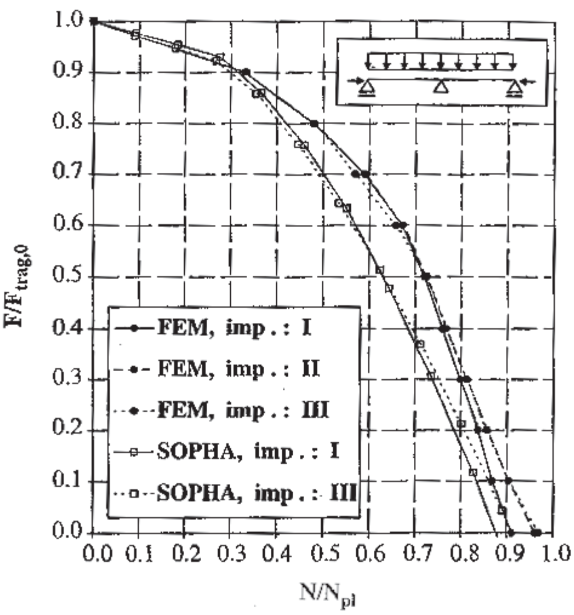


Figure 11. IPE 200, $N + M_z, \overline{\lambda}_z = 0.5$, LC a.

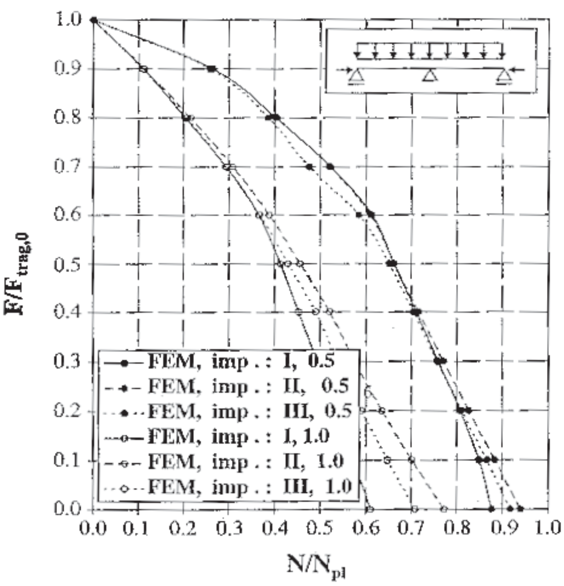


Figure 12. HEB 300, $N + M_z, \overline{\lambda}_z = 0.5$, LC a.

cross section is built up by a set of rectangular cells with four integration points at each cell.

For different slendernesses, load cases and cross sections the results are given in the following figures:

- Load case a, $N + M_y$: Figs. 7–8,
- Load case b, $N + M_y$: Figs. 9–10 and
- Load case a, $N + M_z$: Figs. 11–12.

### 3.2 *Three-span beam*

In the same way some investigations were made for a three-span beam (see Fig. 13), whereby the calculations were carried out with the FEM-method only. The influence of different shapes of initial imperfections can be seen in Fig. 14–Fig. 16.

## 4 CONCLUSIONS

An initial imperfection according to the first buckling mode of type *I* leads in most cases to the smallest ultimate load and therefore this approach must generally be chosen. The results are comparable for

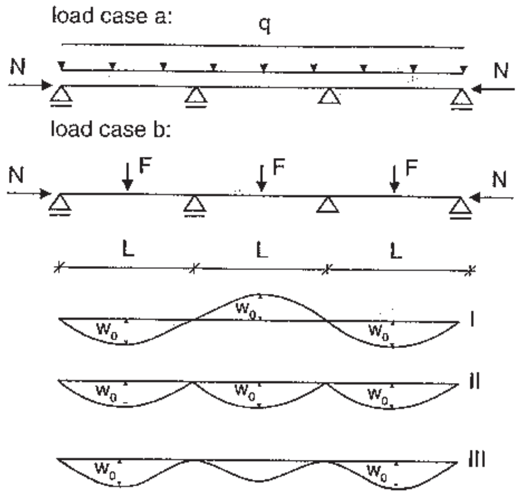


Figure 13. Three-span beam, load cases and imperfections.

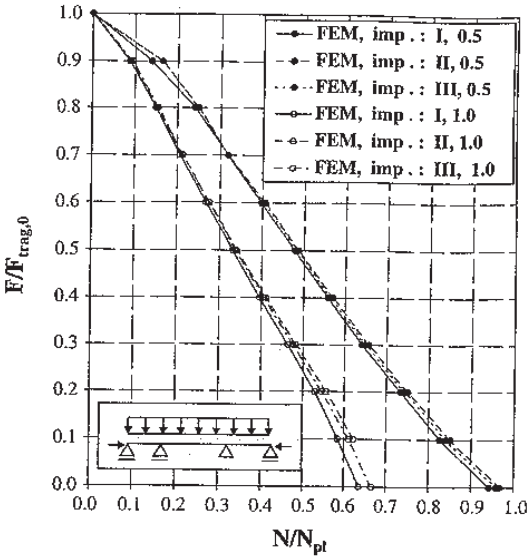


Figure 15. HEB 300, $N + M_y, \overline{\lambda}_y = 0.5$ and 1.0, LC a.

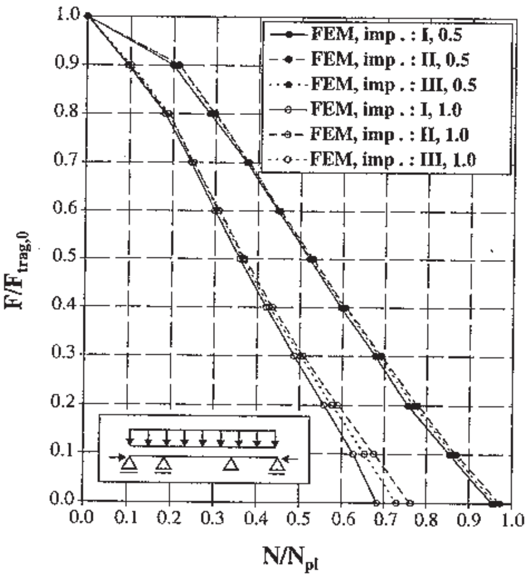


Figure 14. IPE 200, $N + M_y, \overline{\lambda}_y = 0.5$ and 1.0, LC a.

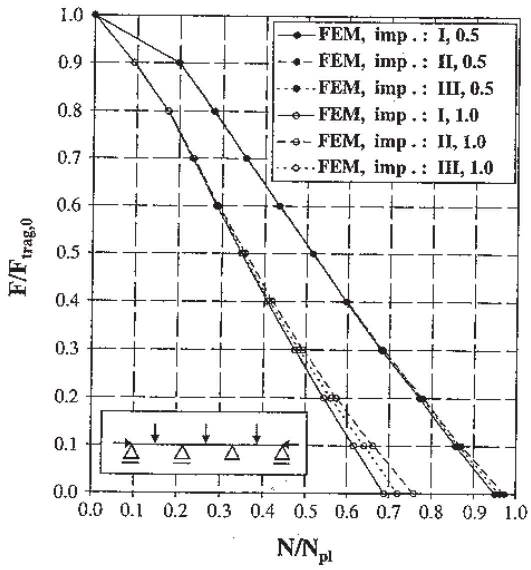


Figure 16. IPE 200, $N + M_y, \overline{\lambda}_y = 0.5$ and 1.0, LC b.

bending moments $M_y$ or $M_z$ as well as for standard and wide flange cross sections. The ultimate load curves for the different imperfections are nearly identical even if the transverse loads are dominant.

Initial imperfections of type *II* and *III* lead to quite similar results as those of type *I*. Only if the transverse load is very small there are some greater differences up to 3% but this is of no importance for practical design because in these cases initial imperfections of type *I* have to be chosen anyway.

As mentioned in chapter 1, the chosen maximum sizes of the initial imperfections according to (DIN 18800-2 1990) are lower bound values. Therefore, in the examples which are dealt with here the calculations according to second order plastic hinge analysis always lead to smaller ultimate loads than the calculation using the FEM method. If the numerical values for the maximum sizes would be chosen in dependency of $\overline{\lambda}$ as recommended in (Eurocode 3 1992) the results would be closer together. But for practical design this dependency on $\overline{\lambda}$ is very unpractical and should be avoided.

Similary investigations will be carried out in the near future for beam columns where the failure by lateral torsional buckling occur.

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